SAMPLE QUESTION PAPER (STANDARD) - 09

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

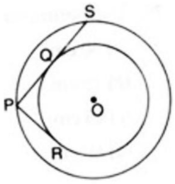
Section A

1. The ratio in which (4, 5) divides the join of (2, 3) and (7, 8) is **[1]**

   a) 2 : 3 b) -3 : 2

   c) -2 : 3 d) 3 : 2

2. The given figure shows two concentric circles with centre O. PR and PQS are tangents to the inner circle from point P lying on the outer circle. If PR = 7.5 cm, then PS is equal to **[1]**



   a) 10 cm. b) 18 cm.

   c) 12 cm. d) 15 cm.

3. Two numbers 'a' and 'b' are selected successively without replacement in that order from the integers 1 to 10. The probability that $\frac{a}{b}$ is an integer, is **[1]**

   a) $\frac{17}{45}$ b) $\frac{8}{45}$

   c) $\frac{1}{5}$ d) $\frac{17}{90}$

4. If the centroid of the triangle formed by the points (3, -5), (-7,4), (10, -k) is at the point (k, -1), then k = **[1]**

   a) 2 b) 1

c) 4 d) 3

5. The value of a so that the point (3, a) lies on the line represented by 2x - 3y = 5 is **[1]**

a) $\frac{1}{3}$ b) – 1

c) 1 d) $\frac{-1}{3}$

6. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively **[1]**

a) (0, – 5) and (2, 0) b) (0, 4) and (– 10, 0)

c) (0, 10) and (– 4, 0) d) (0, – 10) and (4, 0)

7. Two dice are thrown simultaneously. The probability of getting a doublet is **[1]**

a) $\frac{1}{6}$ b) $\frac{1}{3}$

c) $\frac{2}{3}$ d) $\frac{1}{4}$

8. The radius and height of a right circular cone and that of a right circular cylinder are respectively equal. If the volume of the cylinder is 300 cu.cm, then the volume of the cone is **[1]**

a) 900 cu.cm b) 600 cu.cm

c) 100 cu.cm d) 300 cu.cm

9. A lot consists of 40 mobile phones of which 32 are good, 3 have only minor defects and 5 have major defects. Ram will buy a phone if it is good or have minor defects. One phone is selected at random. The probability that it is acceptable to Ram is: **[1]**

a) $\frac{3}{40}$ b) $\frac{4}{5}$

c) $\frac{3}{5}$ d) $\frac{7}{8}$

10. If the product of the roots of the equation x^2 - 3x + k = 10 is -2 then the value of k is **[1]**

a) -8 b) 12

c) -2 d) 8

11. The length of a rectangular field exceeds its breadth by 8 m and the area of the field is 240 m^2. The breadth of the field is **[1]**

a) 16 m b) 30 m

c) 12 m d) 20 m

12. If θ is an acute angle such that $\tan^2\theta = \frac{8}{7}$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is **[1]**

a) $\frac{64}{49}$ b) $\frac{7}{8}$

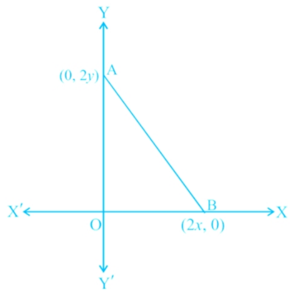
c) $\frac{7}{4}$ d) $\frac{8}{7}$

13. The HCF of 867 and 255 is **[1]**

a) 51 b) 35

c) 25 d) 55

14. The coordinates of the point which is equidistant from the three vertices of a ΔAOB as shown in the figure is **[1]**



a) $\left(\frac{x}{2}, \frac{y}{2}\right)$

b) (y, x)

c) (0, 0)

d) (x, y)

15. The tops of two towers of heights x and y, standing on a level ground subtend angles of 30° and 60° respectively at the centre of the line joining their feet. Then, x : y is **[1]**

a) 1 : 3

b) 2 : 1

c) 1 : 2

d) 3 :1

16. Median = ? **[1]**

a) $l + \left\{h \times \frac{\left(cf - \frac{N}{2}\right)}{f}\right\}$

b) $l - \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

c) $l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

d) none of these

17. LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is **[1]**

a) 560

b) 1120

c) 1680

d) 40

18. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and denominator. If 3 is added to both the numerator and denominator it becomes $\frac{5}{6}$, then the fraction is **[1]**

a) $\frac{9}{7}$

b) $\frac{-9}{7}$

c) $\frac{7}{9}$

d) $\frac{-7}{9}$

19. **Assertion (A):** If a number x is divided by y(x, y) (both x and y are positive) then remainder will be less than x. **[1]**
Reason (R): Dividend = Divisor × Quotient + Remainder.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two right-angled triangles are always similar. **[1]**
Reason (R): By Pythagoras Theorem, $H^2 = P^2 + B^2$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out **[2]**

i. an orange flavoured candy?

ii. a lemon flavoured candy?

22. Taxi charges in a city consist of fixed charges and the remaining depending upon the distance travelled in kilometres. If a person travels 60 km, he pays ₹960, and for travelling 80 km, he pays ₹1260. Find the fixed charges and the rate per kilometre. **[2]**

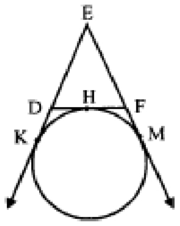
23. If α and β are the zeros of the quadratic polynomial f (x) = $6x^2 + x - 2$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$. **[2]**

24. Let A(4, 2), B (6, 5) and C(1, 4) be the vertices of triangle ABC. Find the coordinates of the point P on AD such that AP : PD = 2:1. **[2]**

OR

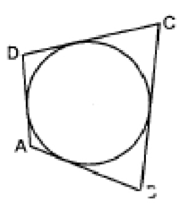
In what ratio does the point C $(\frac{3}{5}, \frac{11}{5})$ divide the line segment joining the points A (3, 5) and B (-3, -2)?

25. In the adjoining figure, a circle touches the side DF of $\triangle$EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then what is perimeter of $\triangle$EDF? **[2]**



OR

In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



Section C

26. If $\cot\theta = \frac{7}{8}$, then evaluate: $\cot^2\theta$ **[3]**

27. Form the pair of linear equations in the problem, and find its solution (if it exists) by the elimination method: Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu? **[3]**

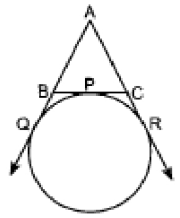
28. Prove that $\frac{1}{\sqrt{2}}$ is irrational. **[3]**

OR

Prove that $\sqrt{5}$ is irrational.

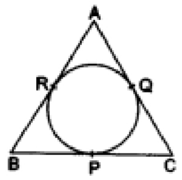
29. In ΔABC, D is the mid-point of BC and ED is the bisector of the $\angle$ADB and EF is drawn parallel to BC cutting AC in F. Prove that $\angle$EDF is a right angle. **[3]**

30. In figure, a circle touches the side BC of $\triangle$ABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of $\triangle$ABC. **[3]**



OR

In the given figure, the incircle-of $\triangle ABC$ touches the sides BC, CA and AB at P, Q and R respectively. Prove that (AR + BP + CQ) = (AQ + BR + CP) $= \frac{1}{2}$(perimeter of ΔABC).



31. A path separates two walls. A ladder leaning against one wall rests at a point on the path. It reaches a height of 90 m on the wall and makes an angle of 60° with the ground. If while resting at the same point on the path, it were made to lean against the other wall, it would have made an angle of 45° with the ground. Find the height it would have reached on the second wall. **[3]**

Section D

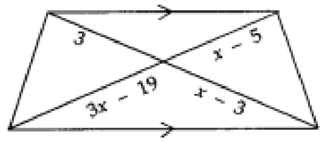
32. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately. **[5]**

OR

Determine whether the given quadratic equation have real roots and if so, find the roots

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

33. If a line is drawn parallel to one side of a trianlge, prove that the other two sides are divided in the same ratio. Using the above result, find x from the adjoining figure. **[5]**

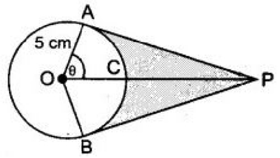


34. A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use π = 3.14) **[5]**

i. minor sector
ii. major sector
iii. minor segment
iv. major segment

OR

An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.



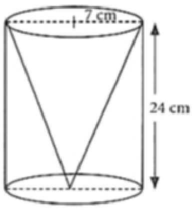
35. Find the mean of the following frequency distribution: **[5]**

Class interval	0-8	8-16	16-24	24-32	32-40
Frequency	6	7	10	8	9

Section E

36. **Read the text carefully and answer the questions:** **[4]**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod’s mind.



(i) Find the slant height of the conical cavity so formed?

(ii) Find the curved surface area of the conical cavity so formed?

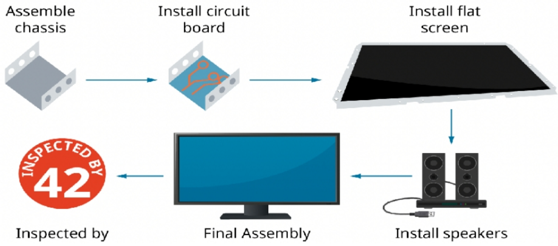
(iii) Find the external curved surface area of the cylinder?

OR

Find the ratio of curved surface area of cone to curved surface area of cylinder?

37. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year.



Assuming that production increases uniformly by a fixed number every year.

(i) Find the production in the 1st year.

(ii) Find the production in the 5th year.

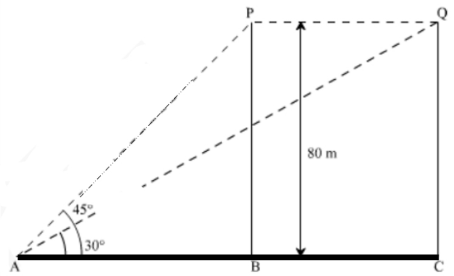
OR

Find in which year 10000 units are produced?

(iii) Find the total production in 7 years.

38. **Read the text carefully and answer the questions:** **[4]**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.



(i) Find the distance between observer and the bottom of the tree?

(ii) Find the speed of the bird?

OR

Find the distance between initial position of bird and observer?

(iii) Find the distance between second position of bird and observer?

Solution

SAMPLE QUESTION PAPER (STANDARD) - 09

Class 10 - Mathematics

Section A

1. **(a)** 2 : 3

   Explanation: Let the point (4, 5) divides the line segment joining the points (2, 3) and (7, 8) in the ratio m: n

   $\therefore 4 = \frac{mx_2+nx_1}{m+n} = \frac{m\times7+n\times2}{m+n}$

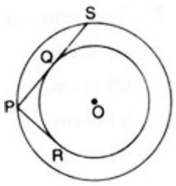
   $\Rightarrow$ 4(m + n) = 7m + 2n $\Rightarrow$ 4m + 4n = 7m + 2n

   4n - 2n = 7m - 4m

   $\Rightarrow$ 2n = 3m $\Rightarrow \frac{m}{n} = \frac{2}{3}$

   $\therefore$ m : n = 2 : 3

2. **(d)** 15 cm.

   Explanation: 

   Construction: Joined OP and OS. Draw OQ $\perp$ PS

   Here PR = PQ = 7.5 cm [Tangents to a circle from an external point]

   Now, in triangles OPQ and OSQ,

   $\angle$PQO = $\angle$SQO = 90° [Line segment is perpendicular from centre to point of contact]

   OQ = OQ [Common]

   OP = OS [Radii]

   $\therefore \Delta$ OPQ $\cong \Delta$ OSQ [SAS congruency]

   $\therefore$ PQ = QS [By CPCT]

   Therefore, PS = PQ + QS

   = 7.5 + 7.5 = 15 cm

3. **(d)** $\frac{17}{90}$

   Explanation: a and b are two number to be selected from the integers = 1 to 10 without replacement of a and b

   i.e., 1 to 10 = 10

   and 2 to 10 = 9

   No. of ways = 10 $\times$ 9 = 90

   Probability of $\frac{a}{b}$ where it is an integer

   $\therefore$ Possible event will be

   $= \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{6}{1}, \frac{7}{1}, \frac{8}{1}, \frac{9}{1}, \frac{10}{1}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}, \frac{10}{2}, \frac{6}{3}, \frac{9}{3}, \frac{8}{4}, \frac{10}{5}$ = 17

   $P(E) = \frac{m}{n} = \frac{17}{90}$

4. **(a)** 2

   Explanation: O(k, -1) is the centroid of triangle whose vertices are

   A(3, -5), B(-7, 4), C(10, -k)

   $\therefore k = \frac{x_1+x_2+x_3}{3}$

   $\Rightarrow k = \frac{3-7+10}{3} = \frac{6}{3} = 2$

5. **(a)** $\frac{1}{3}$

   Explanation: 2x - 3y = 5

   $\Rightarrow$ 2 $\times$ 3 - 3 $\times$ a = 5

   $\Rightarrow$ 6 - 3a = 5

   $\Rightarrow$ a = $\frac{1}{3}$

6. **(d)** (0, – 10) and (4, 0)

   Explanation:

Let the coordinates of P (0, y) and Q (x, 0).

So, the mid - point of P (0, y) and Q (x, 0) = M

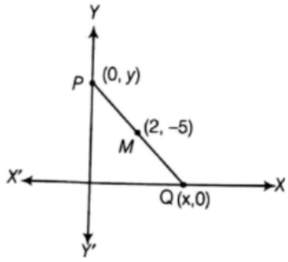
Coordinates of M = $\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

$\therefore$ Mid - point of a line segment having points (x_1, y_1) and (x_2, y_2)

= $\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$

Given,

Mid - point of PQ is (2, - 5)



$\therefore 2 = \frac{x+0}{2}$ = 4 = x + 0

x = 4

$-5 = \frac{y+0}{2}$ = - 10 = y + 0

- 10 = y

So,

x = 4 and y = - 10

Thus, the coordinates of P and Q are (0, - 10) and (4, 0)

7. **(a)** $\frac{1}{6}$

Explanation: Doublet means getting same number on both dice simultaneously

Doublets = (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Number of possible outcomes = 6

Total number of ways to throw a dice = 36

Probability of getting a doublet = $\frac{6}{36} = \frac{1}{6}$

8. **(c)** 100 cu.cm

Explanation: Let the Radius and height of the cone be 'r' and 'h' respectively and Radius and height of the right circular cylinder be R and H.

$\therefore$ The volume of Cylinder = $\pi R^2 H$ = 300 cu. cm

And Volume of Cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi R^2 H$

= $\frac{1}{3} \times 300$

= 100 cu. cm

9. **(d)** $\frac{7}{8}$

Explanation: Number of phones which are good and minor defects = 32 + 3 = 35

Number of possible outcomes = 35

Number of Total outcomes = 40

$\therefore$ Required Probability = $\frac{35}{40} = \frac{7}{8}$

10. **(d)** 8

Explanation: Given equation is x^2 - 3x + (k - 10) = 0.

Product of roots = (k - 10).

So, k - 10 = -2 $\Rightarrow$ k = 8.

11. **(c)** 12 m

Explanation: Let the breadth of the rectangular field be x m

Therefore Length of the rectangular field = (x + 8) m

Area of the rectangular field = 240 m^2 (Given)

$\therefore (x+8) \times x = 240$ (Area = Length $\times$ Breadth)

$\Rightarrow x^2 + 8x - 240 = 0$

$\Rightarrow x(x+20)-12(x+20)=0$
$\Rightarrow (x+20)(x-12)=0$
$\Rightarrow x+20=0$ or x - 12 = 0
$\Rightarrow$ x = -20 or x = 12
$\therefore$ x = 12 (Breadth cannot be negative)
Thus the breadth of the field is 12 m.

12. **(b)** $\frac{7}{8}$

Explanation: $\tan^2\theta=\frac{8}{7}\Rightarrow\tan\theta=\frac{\sqrt{8}}{\sqrt{7}}=\frac{2\sqrt{2}}{\sqrt{7}}=\frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras Theorem

$(\text{Hyp.})^2 = (\text{Base})^2 + (\text{Perp.})^2$

$=(\sqrt{7})^2+(2\sqrt{2})^2=7+8=15$

$\therefore \text{Hyp.}=\sqrt{15}$

$\therefore \sin\theta=\frac{\text{Perpendicular}}{\text{Hypotenuse}}=\frac{2\sqrt{2}}{\sqrt{15}}$

$\cos\theta=\frac{\text{Base}}{\text{Hypotenuse}}=\frac{\sqrt{7}}{\sqrt{15}}$

Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$=\frac{1-\sin^2\theta}{1-\cos^2\theta}=\frac{1-\left(\frac{2\sqrt{2}}{\sqrt{15}}\right)^2}{1-\left(\frac{\sqrt{7}}{\sqrt{15}}\right)^2}$

$=\frac{1-\frac{8}{15}}{1-\frac{7}{15}}=\frac{\frac{15-8}{15}}{\frac{15-7}{15}}=\frac{\frac{7}{15}}{\frac{8}{15}}$

$=\frac{7}{15}\times\frac{15}{8}=\frac{7}{8}$

13. **(a)** 51

Explanation: 867 = 255 × 3 + 102

255 = 102 × 2 + 51

102 = 51 × 2 + 0

Hence, we got remainder as 0 , therefore HCF of (867 , 255) is 51

14. **(d)** (x, y)

Explanation: AB = $\sqrt{(2x-0)^2+(0-2y)^2}$

= $\sqrt{4x^2+4y^2}$ = $2\sqrt{x^2+y^2}$ units

BO = $\sqrt{(0-2x)^2+(0-0)^2}$

= $\sqrt{4x^2}$ = 2x units

AO = $\sqrt{(0-0)^2+(0-2y)^2}$

= $\sqrt{4y^2}$ = $2y$ units

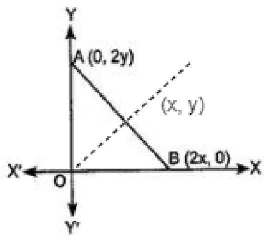
Now, $\text{AB}^2=\text{AO}^2+\text{BO}^2\Rightarrow\left(2\sqrt{x^2+y^2}\right)^2=(2x)^2+(2y)^2$

$\Rightarrow 4\left(x^2+y^2\right)=4\left(x^2+y^2\right)$

Therefore, triangle AOB is an isosceles right-angled triangle.

Since the coordinate of the point which is equidistant from the three vertices of a right-angled triangle is the coordinates of mid-point of its hypotenuse.

$\therefore$ Mid-point of AB = $\left(\frac{0+2x}{2},\frac{2y+0}{2}\right)$ = (x,y)



15. **(a)** 1 : 3

Explanation: Let AB and CD be the given pillars and O be the midpoint of AC.

Then, AB = x, CD = y, ∠AOB = 30° and ∠COD = 60°.

From right △OAB, we have

$\frac{OA}{AB} = \cot 30^\circ \Rightarrow \frac{OA}{x} = \sqrt{3}$

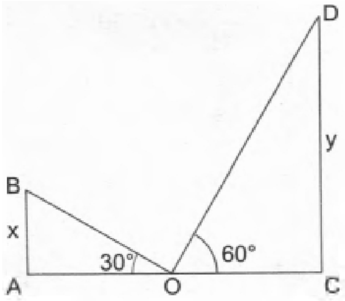
$\Rightarrow$ OA = $x\sqrt{3}$. .....(i)

From right △OCD, we have

$\frac{OC}{CD} = \cot 60^\circ \Rightarrow \frac{OC}{y} = \frac{1}{\sqrt{3}} \Rightarrow OC = \frac{y}{\sqrt{3}}$ ....(ii)

But, OA = OC

$\therefore x\sqrt{3} = \frac{y}{\sqrt{3}} \Rightarrow 3x = y \Rightarrow \frac{x}{y} = \frac{1}{3} \Rightarrow$ x : y = 1 : 3



16. **(c)** $l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

17. **(c)** 1680

Explanation: LCM = Product of greatest power of each prime factor involved in the numbers

= $2^4 \times 3 \times 5 \times 7$

= $16 \times 3 \times 5 \times 7$

= 1680

18. **(c)** $\frac{7}{9}$

Explanation: Let the fraction be $\frac{x}{y}$.

According to question

$\frac{x+2}{y+2} = \frac{9}{11}$

$\Rightarrow$ 11x + 22 = 9y + 18

$\Rightarrow$ 11x - 9y = -4 ... (i)

And $\frac{x+3}{y+3} = \frac{5}{6}$

$\Rightarrow$ 6x + 18 = 5y + 15

$\Rightarrow$ 6x - 5y = -3 ... (ii)

On solving eq. (i) and eq. (ii), we get

x = 7, y = 9

Therefore, the fraction is $\frac{7}{9}$

19. **(d)** A is false but R is true.

Explanation: Remainder is less than by divisor not by divident.

20. **(d)** A is false but R is true.

Explanation: Two right-angled triangle are similar only if at least one more angle is also equal.

Section B

21. i. The probability that she takes out an orange flavoured candy is 0 because the bag contains lemon flavoured candies only.

ii. Probability that she takes out a lemon flavoured candy is 1 because the bag contains lemon flavoured candies only.

22. Let the fixed charges be ₹ x and the other charges be ₹ y per km.

As per given condition,

If a person travels 60 km, he pays ₹ 960

Then, x + 60y = 960 .... (i)

And for travelling 80 km, he pays ₹ 1260.

Then, x + 80y = 1260. .... (ii)

On subtracting (i) from (ii), we get

$20y = 300$

$\Rightarrow y = \frac{300}{20}$

$\Rightarrow$y = 15

Putting y = 15 in (i), we get

x + 60y = 960

x + (60 $\times$ 15) = 960

$\Rightarrow$ x = 960 - 900

$\Rightarrow$ x = 60.

Therefore, fixed charges = ₹ 60 and the rate per km = ₹ 15 per km.

23. Since, α and β are the zeroes of the quadratic polynomial f(x) = 6x^2 + x - 2.

a=6, b=1, c=-2

sum of zeros =α+β=$-\frac{b}{a}=\frac{-1}{6}$

Product of the zeroes = αβ=$\frac{c}{a}=\frac{-1}{3}$

Now,

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\left(-\frac{1}{6}\right)^2-2(-\frac{1}{3})}{-\frac{1}{3}}=\frac{\frac{1}{36}+\frac{2}{3}}{-\frac{1}{3}}=-\frac{\frac{75}{108}}{\frac{1}{3}}=-\frac{75}{108}\times\frac{3}{1}=-\frac{225}{108}=-\frac{25}{12}$$

24. AD is the median of triangle ABC

∴ D is the mid-point of BC.

∴ Its coordinates are $(\frac{7}{2},\frac{9}{2})$

$P \to \left(\frac{(2)\left(\frac{7}{2}\right)+(1)(4)}{2+1},\frac{(2)\left(\frac{9}{2}\right)+(1)(2)}{2+1}\right)$ [Using section formula]

$\Rightarrow P \to \left(\frac{11}{3},\frac{11}{3}\right)$

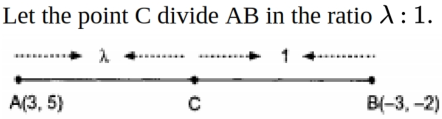
$E \to \left(\frac{4+1}{2},\frac{2+4}{2}\right)$

$\Rightarrow E \to \left(\frac{5}{2},3\right)$

OR

According to the question, A(3, 5) and B (-3, -2)

Let the point C divide AB in the ratio $\lambda : 1$.



By using section formula, $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

The coordinates of C are

$\left(\frac{-3\lambda+3}{\lambda+1},\frac{-2\lambda+5}{\lambda+1}\right)$

But, the coordinates of C are given as $(\frac{3}{5},\frac{11}{5})$

$\therefore \quad \frac{-3\lambda+3}{\lambda+1}=\frac{3}{5}$ and $\frac{-2\lambda+5}{\lambda+1}=\frac{11}{5}$

$\Rightarrow \quad -15\lambda+15=3\lambda+3$ and $-10\lambda+25=11\lambda+11$

$\Rightarrow \quad 18\lambda=12$ and $21\lambda=14$

$\Rightarrow \quad \lambda=\frac{2}{3}$

Hence, the point C divides AB in the ratio 2 : 3.

25. We know that tangent segments to a circle from the same external point are Equal. Therefore, we have

EK = EM = 9 cm

Now, EK + EM = 18 cm

$\Rightarrow ED+DK+EF+FM=18cm$

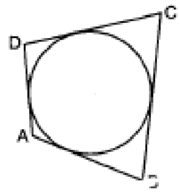
$\Rightarrow ED+DH+EF+HF=18cm$

$\Rightarrow ED+DF+EF=18cm$

$\Rightarrow$ Perimeter of $\triangle$EDF $=18cm$

OR

Given, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm.



If a circle touches all the four sides of quadrilateral ABCD, then

AB + CD = AD + BC

$\therefore$ 6 + 8 = AD + 9

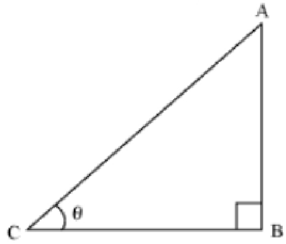
$\Rightarrow$ 14 = AD + 9

$\Rightarrow$ 14-9 = AD

$\Rightarrow$ AD = 5 cm

Section C

26. Let us consider a right angled ΔABC right angled at point B.



Let $\angle C = \theta$

Given,

$\cot\theta = \frac{7}{8} = \frac{\textit{Side adjacent to } \angle\theta}{\textit{Side opposite to } \angle\theta} = \frac{\text{BC}}{\text{AB}}$

If BC is 7K then AB will be 8K, where K is a positive integer.

Now applying Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

Or, $AC^2 = 64K^2 + 49K^2$

Or, $AC^2 = 113K^2$

$\therefore \text{AC} = \sqrt{113}\text{K}$

Now,

$\sin\theta = \frac{\textit{Side opposite to } \angle\theta}{\textit{hypotenuse}} = \frac{\text{AB}}{\text{AC}}$

$= \frac{8K}{\sqrt{113}K} = \frac{8}{\sqrt{113}}$

And,

$\cos\theta = \frac{\textit{Side adjacent to } \angle\theta}{\textit{hypotenuse}} = \frac{\text{BC}}{\text{AC}}$

$= \frac{7K}{\sqrt{113}K} = \frac{7}{\sqrt{113}}$

Now $\cot^2\theta = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{7}{\sqrt{113}} \times \frac{\sqrt{113}}{8}\right)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

27. Let the present age of Nuri and Sonu be x years and y years respectively.

Then, according to the question,

x – 5 = 3(y – 5)

$\Rightarrow$ x – 5 = 3y – 15

$\Rightarrow$x – 3y = –10 .... (1)

x + 10 = 2(y + 10)

x + 10 = 2y + 20

$\Rightarrow$ x – 2y = 10 ------------ (2)

Subtracting equation (2) from equation (1), we get

– y= – 20

$\Rightarrow$ y = 20

Subtracting equation (2) from equation (1), we get

x – 2(20) = 10

$\Rightarrow$ x – 40 = 10

$\Rightarrow$ x = 40 + 10

$\Rightarrow$ x = 50

Hence, Nuri and Sonu are 50 years and 20 years old respectively at present.

Verification.Subtracting the value of x = 50 and y = 20,we find that both the eqations (1) and (2) are satisfied as shown below:

x – 3y = 50 – 3(20) = 50 – 60 = – 10

x – 2y = 50 – 2(20) = 50 – 40=10

Hence,the solution is correct.

28. We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$is rational.

It means we have some co-prime integers *a* and *b* ($b \neq 0$)

Such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$ ....(1)

R.H.S of (1) is rational but we know that is$\sqrt{2}$ irrational.

It is not possible which means our supposition is wrong.

Therefore,$\frac{1}{\sqrt{2}}$can not be rational.

Hence, it is irrational.

OR

Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers *a* and b ($b \neq 0$)

Such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow b\sqrt{5}=a$

Squaring both sides, we get

$\Rightarrow 5b^2 = a^2$ ... (1)

It means that 5 is factor of a^2

Hence, 5 is also factor of *a* by Theorem. ... (2)

If, 5 is factor of *a* , it means that we can write $a = 5c$ for some integer *c* .

Substituting value of *a* in (1) ,

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

It means that 5 is factor of b^2 .

Hence, 5 is also factor of *b* by Theorem. ... (3)

From (2) and (3) , we can say that 5 is factor of both *a* and *b* .

But, *a* and *b* are co-prime .

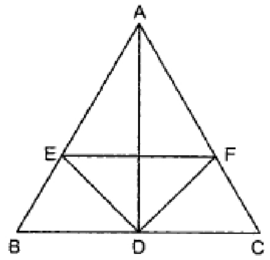
Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

29. **GIVEN :** ΔABC in which D is the mid-point of side BC and ED is the bisector of $\angle$ADB, meeting AB in E. EF is drawn parallel to BC meeting AC in F.

TO PROVE :$\angle$EDF is a right angle.

PROOF: In ΔADB, DE is the bisector of $\angle$ADB.

Since the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.



$\therefore \quad \frac{AD}{DB} = \frac{AE}{EB}$

$\Rightarrow \quad \frac{AD}{DC} = \frac{AE}{EB}$ [$\because$ D is the mid-point of BC $\therefore$ DB = DC] .....(i)

In ΔABC, we have,

$EF \| BC$

Therefore, by basic proportionality theorem,we have,

$\frac{AE}{EB} = \frac{AF}{FC}$ ..... (ii)

From (i) and (ii), we get

$\frac{AD}{DC} = \frac{AF}{FC}$

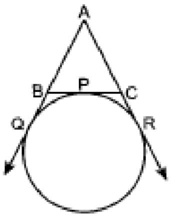
$\Rightarrow$ In ΔADC, DF divides AC in the ratio AD: DC

$\Rightarrow$ DF is the bisector of $\angle$ADC

Thus, DE and DF are the bisectors of adjacent supplementary angles $\angle$ADB and $\angle$ADC respectively.

Hence, $\angle$EDF is a right angle.

30. Given:



AQ = 5 cm.

To find: Perimeter of $\triangle$ABC.

Sol.: AQ and AR are tangents from the same point

AQ = AR = 5 cm ...(i)[Tangents from the same external points are equal]

BQ and BP are tangents from same point

$\Rightarrow$ BQ = BP (ii)

CP and CR are also tangents from the same point

$\Rightarrow$ CP = CR (iii)

In $\triangle$ABC, Perimeter of $\triangle$ ABC= $AB + BC + AC$

= AB + BP + CP + AC

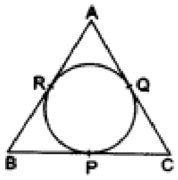
$AB + BQ + CR + AC = AQ + AR$ [From (ii) and (iii)]

= 5 cm + 5 cm

= 10 cm [From (i)]

Perimeter of $\triangle$ ABC = 10 cm

OR



We know that the lengths of tangents from an exterior point to a circle are equal.

$\therefore$ AR = AQ, ... (i) [tangents from A]

BP = BR, ... (ii) [tangents from B]

CQ = CP .... (iii) [tangents from C]

$\therefore$ (AR + BP + CQ) = (AQ + BR + CP) = k (say).

Perimeter of $\triangle ABC$ = (AB + BC + CA)

= (AR + BR) + (BP + CP) + (CQ+AQ)

= (AR + BP + CQ) + (AQ + BR + CP)

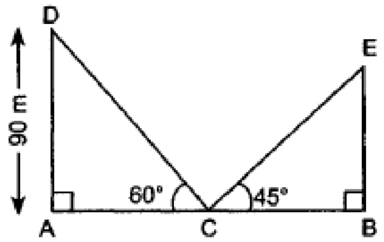
= (k + k) = 2k

$\Rightarrow \quad k = \frac{1}{2}(\text{ perimeter of } \triangle ABC)$.

$\therefore$ (AR + BP +CQ) = (AQ + BR + CP)

$= \frac{1}{2}(\text{ perimeter of } \triangle ABC)$.

31. Let AB is path



In rt. △DAC, $\frac{DC}{AD} = cosec\, 60^\circ$

$\Rightarrow \frac{DC}{90} = \frac{2}{\sqrt{3}}$

$DC = \frac{2}{\sqrt{3}} \times 90m = \frac{180}{\sqrt{3}}m$

Now, $DC = CE$

$\therefore \quad CE = \frac{180}{\sqrt{3}}m$

In rt. △EBC,

$\frac{BE}{CE} = \sin 45^\circ$

$\Rightarrow \quad BE = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}}m$

$\Rightarrow \quad BE = 73.47m$

Section D

32. Let time taken by pipe A be x minutes, and time taken by pipe B be x + 5 minutes.

In one minute pipe A will fill $\frac{1}{x}$ tank

In one minute pipe B will fill $\frac{1}{x+5}$ tank

pipes A + B will fill in one minute = $\frac{1}{x} + \frac{1}{x+5}$ tank

Now according to the question.

$\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$

or, $\frac{x+5+x}{x(x+5)} = \frac{9}{100}$

or, 100(2x + 5) = 9x(x + 5)

or, $200x + 500 = 9x^2 + 45x$

or, $9x^2 - 155x - 500 = 0$

or, $9x^2 - 180x + 25x - 500 = 0$

or,9x(x - 20) + 25(x - 20) = 0

or, (x-20)(9x + 25) = 0

or, $x = 20, \frac{-25}{9}$

rejecting negative value, x = 20 minutes

and x + 5 = 25 minutes

Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

OR

We have,

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Here, $a = \sqrt{3}, b = 10$ and $c = -8\sqrt{3}$

$\therefore D = b^2 - 4ac$

$= (10)^2 - 4 \times (\sqrt{3}) \times (-8\sqrt{3})$

$= 100 + 96 = 196 > 0$

$\therefore D > 0$

So, the given equation has real roots, given by

$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-10+\sqrt{196}}{2\times\sqrt{3}}$

$= \frac{-10+14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$

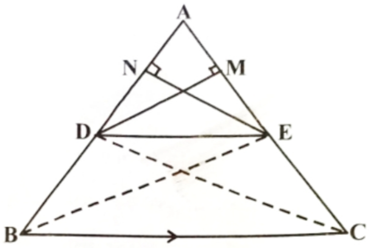
And, $\beta = \frac{-b-\sqrt{D}}{2a}$

$= \frac{-10-\sqrt{196}}{2\sqrt{3}}$

$= \frac{-10-14}{2\sqrt{3}} = -\frac{24}{2\sqrt{3}} = -\frac{12}{\sqrt{3}}$

$= -\frac{4\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = -4\sqrt{3}$

$\therefore\ x = \frac{2}{\sqrt{3}}, -4\sqrt{3}$

33. 

Given: $\triangle$ABC in which DE||BC

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and DC and Draw EN$\perp$AD and DM $\perp$AE.

Proof: Consider $\triangle$DBE,

Area of $\triangle$DBE=$\frac{1}{2}$DB$\times$EN .....(i)

Again, Consider $\triangle$ADE

Area of $\triangle$ADE= $\frac{1}{2}$AD$\times$EN.....(ii)

Divide eq. (i) by (ii), we get,

$\Rightarrow \frac{Area(\Delta DBE)}{Area(\Delta ADE)} = \frac{\frac{1}{2}\times DB\times EN}{\frac{1}{2}\times AD\times EN}$

$\Rightarrow \frac{Area(\Delta DBE)}{Area(\Delta ADE)} = \frac{DB}{AD}$ .....(iii)

Now Consider $\triangle$ECD,

Area of $\triangle$ECD=$\frac{1}{2}$EC$\times$DM ...(iv)

Again, Consider $\triangle$ADE

Area of $\triangle$ADE= $\frac{1}{2}$AE$\times$DM...(v)

Divide eq. (iv) by (v), we get,

$\Rightarrow \frac{Area(\Delta ECD)}{Area(\Delta ADE)} = \frac{\frac{1}{2}\times EC\times DM}{\frac{1}{2}\times AE\times DM}$

$\Rightarrow \frac{Area(\Delta ECD)}{Area(\Delta ADE)} = \frac{EC}{AE}$ ......(vi)

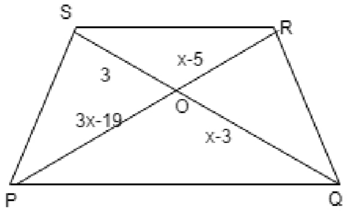
Since $\triangle$DBE and $\triangle$EDC lies in the same base i.e. DE and the same parallels i.e. DE and BC.

$\Rightarrow$ Area ($\triangle$DBE)= Area ($\triangle$EDC)

From (iii) and (vi), we get

$\frac{DB}{AD}=\frac{EC}{AE}$

Hence Proved



Since,SR||PQ, $\Rightarrow \Delta POQ \sim \Delta ROS$ [By AAsimilarty criteria]

$\Rightarrow \frac{PO}{OR} = \frac{OQ}{OS}$

$\Rightarrow \frac{3x-19}{x-5} = \frac{x-3}{3}$

$\Rightarrow$ 3(3x - 19) = (x-5) (x-3)

$\Rightarrow$ 9x - 57 = x^2 - 8x + 15

$\Rightarrow$ x^2 - 8x - 9x + 15 + 57 = 0

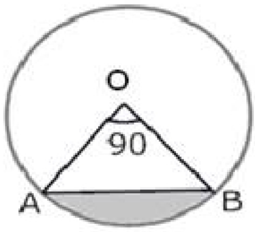
$\Rightarrow$ x^2 - 17x + 72 = 0

$\Rightarrow$ x^2 - 8x - 9x + 72 = 0

$\Rightarrow$ x(x - 8) -9(x - 8) = 0

$\Rightarrow$ (x - 8) (x - 9) = 0

$\Rightarrow$ x = 8 or x = 9

34. 

i. Area of minor sector = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

= 78.50 = 78.5 cm^2

ii. Area of major sector = Area of circle - Area of minor sector

= $\pi(10)^2$ - $\frac{90}{360}$ $\pi(10)^2$ = 3.14 (100) - $\frac{1}{4}$(3.14) (100)

= 314 - 78.50 = 235.5 cm^2

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle$OAB $= \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\,(\because \angle AOB = 90^\circ)$

Area of sector = $\frac{\theta}{360}\pi r^2$

= $\frac{1}{4}$(3.14) (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 cm^2

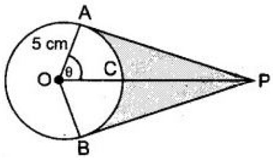
iv. Area of major segment = Area of the circle - Area of minor segment

= $\pi(10)^2$ - 28.5

= 100(3.14) - 28.5

= 314 - 28. 5 = 285.5 cm^2

OR



$\cos\theta = \frac{1}{2}$ or, $\theta = 60^\circ$

Reflex $\angle AOB = 120^\circ$

$\therefore$ ADB = $\frac{2\times3.14\times5\times240}{360}$ = 20.93 cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

a ($\triangle$OAP) = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

Area ($\triangle$OAP + $\triangle$OBP) = 2$\times$ $\frac{25\sqrt{3}}{2} = 25\sqrt{3}$ = 43.25 cm^2

Area of sector OACB = $\frac{\theta}{360} \times \pi r^2$

= $\frac{25\times3.14\times120}{360}$ = 26.16 cm^2

Shaded Area = 43.25 - 26.16 = 17.09 cm^2

35.

Class interval	**Mid- value**	**$d_i = x_i - 20$**	$u_i = \left(\frac{x_i-20}{8}\right)$	**f_i**	**f_iu_i**
0-8	4	-16	-2	6	-12
8-16	12	-8	-1	7	-7
16-24	20	0	0	10	0
24-32	28	8	1	8	8

32-40	36	16	2	9	18
				N = 40	$\sum f_i u_i$ = 7

Let the assumed mean (A) is = 20

h = 8

Mean= A + h$\left(\frac{\sum f_i u_i}{N}\right)$

= 20 + 8$\left(\frac{7}{40}\right)$

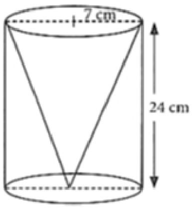
= 20 + 1.4

= 21.4

Section E

36. **Read the text carefully and answer the questions:**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Given height of cone = 24cm and radius of base = r = 7cm

Slant height of conical cavity,

l = $\sqrt{h^2 + r^2}$

= $\sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625}$ = 25 cm

(ii) we know that r = 7cm, l = 25 cm

Curved surface area of conical cavity = πrl

= $\frac{22}{7} \times 7 \times 25$ = 550 cm^2

(iii)For cylinder height = h = 24cm, radius of base = r = 7cm

External curved surface area of cylinder

= 2πrh = 2 $\times \frac{22}{7} \times 7 \times 24$ = 1056 cm^2

OR

Curved surface area of conical cavity = πrl

= $\frac{22}{7} \times 7 \times 25$ = 550 cm^2

External curved surface area of cylinder

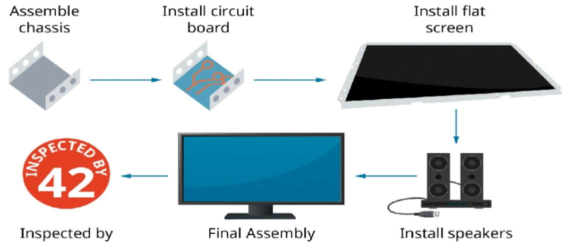
= 2πrh = 2 $\times \frac{22}{7} \times 7 \times 24$ = 1056 cm^2

$\frac{\text{curved surface area of cone}}{\text{curved surface area of cylinder}} = \frac{550}{1056} = \frac{275}{528}$

hence required ratio = 275:528

37. **Read the text carefully and answer the questions:**

Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year.



Assuming that production increases uniformly by a fixed number every year.

(i) Let production in a 1st year be 'a' unit and increase in production (every year) be 'd' units.

Increase in production is constant, therefore unit produced every year forms an AP.

Now, a_3 = 6000

$a + 2d = 6000 \Rightarrow a = 6000 - 2d$ ...(i)

and $a_7 = 7000 \Rightarrow a + 6d = 7000$

$\Rightarrow (6000 - 2d) + 6d = 7000 \Rightarrow 4d = 1000$ [using eq. (i)]

$\Rightarrow d = 250$

When d = 250, eq. (i) becomes

$a = 6000 - 2(250) = 5500$

$\therefore$ Production in 1st year = 5500

(ii) Production in fifth year

$a_5 = a + 4d = 5500 + 4(250) = 6500$

OR

$a_n = 1000$ units

$a_n = 1000$

$\Rightarrow 10000 = a + (n - 1)d$

$\Rightarrow 1000 = 5500 + 250n - 250$

$\Rightarrow 10000 - 5500 + 250 = 250n$

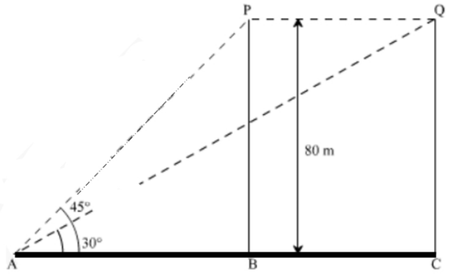
$\Rightarrow 4750 = 250n$

$\Rightarrow n = \frac{4750}{250} = 19$

(iii)Total production in 7 years = $\frac{7}{2}(5500 + 7000) = 43750$

38. **Read the text carefully and answer the questions:**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^o. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30^o. Find the speed of flying of the bird.



(i) Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree

In $\triangle ABP$

$\tan 45^o = \frac{BP}{AB}$

$\Rightarrow 1 = \frac{80}{AB}$

$\Rightarrow AB = 80$ m

(ii) The speed of the bird

In $\triangle AQC$

$\tan 30^o = \frac{QC}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow AC = 80\sqrt{3}$ m

AC - AB = BC

$\Rightarrow BC = 80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$m

Speed of bird = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3} - 1)$

$\Rightarrow$ Speed of the bird = 29.28 m/sec

OR

The distance between initial position of bird and observer.

In $\triangle ABP$

$\sin 45^o = \frac{BP}{AP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{80}{AP}$

$\Rightarrow$ AP = $80\sqrt{2}m$

(iii)The distance between second position of bird and observer.

In $\triangle$AQC

sin 30^o = $\frac{QC}{AQ}$

$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$

$\Rightarrow$ AQ = 160 m